Apparent Cosmic Acceleration via Backreaction from the Causal Propagation of Inhomogeneity Information

("Causal Backreaction")

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It's a Matter of Taste: *Different Methods* for Explaining the (Apparent?) Cosmic Acceleration

Broadly Speaking, there are ~~3~~ 4! principal approaches:

(1) Dark Energy:

(i) Cosmological Constant (ΛCDM) ➔ "Cosmological Constant Problem" & "Coincidence Problem"

(ii) Dynamical Dark Energy (DDE) ➔ *Yet Another* Exotic Substance? (Nonadiabatic Pressure to stay smooth?)

(2) Modified Gravity: f(R) Theories, etc. ➔ "Inelegant" modifications to G.R.?

(3) Inhomogeneities: "Dressed" Cosmological Parameters ➔ Non-Copernican?

(4) Structure formation: *Backreaction on a(t)!* ➔ Strong enough to work…???

Each approach has its own advantages… and its own problems…

After much **subtle, sophisticated debate**:

➔ *Everyone chooses their own favorite approach, anyway!*

Backreaction seems like the perfect solution!

… Automatically triggers at the right time, & with strength based on mass density Ω_M …

➔ No coincidences! No changes to Einstein's G.R.! No special observers!

So why are the "naysayers" saying "Nay"?

Can the Acceleration of Our Universe Be Explained by the Effects of Inhomogeneities?

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Abstract

No. It is simply not plausible that cosmic acceleration could arise within the context of general relativity from a back-reaction effect of inhomogeneities in our universe, without the presence of a cosmological constant or "dark energy." We point out that our universe appears to be described very accurately on all scales by a Newtonianly perturbed FLRW metric. (This assertion is entirely consistent with the fact that we commonly encounter $\delta\rho/\rho > 10^{30}$.) If the universe is accurately described by a Newtonianly perturbed FLRW metric, then the back-reaction of inhomogeneities on the dynamics of the universe is negligible. If not, then it

- The Central Issue: In a Universe with (mostly) Nonrelativistic matter and (mostly) Newtonian Perturbations, how to get a ***Strong-G.R.*** effect like Cosmic Acceleration?

Necessary Physics *(as we'll see…)*

Overlapping/Cumulative summing of pert's. from different inhomogeneities (esp. from *outside* the "local matter horizon")

Vorticity (and/or Velocity Dispersion)
…generated from…
Structure Self-Stabilization & **Virialization**

Causal Gravitational Info Propagation via:
- Terms at least up to $O(v^2)$
- Tensor Components
- "Magnetic" Gravitational Terms

Metric Pert. Potential **Time-Derivatives**

"**Newtonian-Level**" Strength Perturbations

A Dynamical **Phase Transition** from "Smooth" to "Clumped"

Formalisms/Models Lacking It*

(* Or, "How to offend every other researcher in Backreaction")

Backreaction No-Go "Proofs"

Swiss-Cheese Models *(Any Interior)*

Purely Observational "Apparent" Accel. *(e.g., Voids, Lensing effects…)*

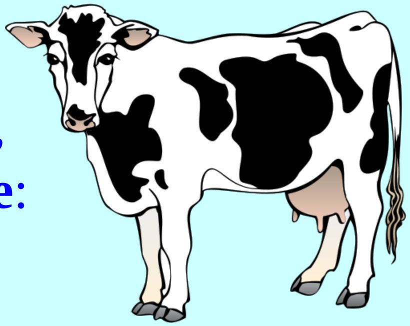
Perturbation Theory Expansions *(Most to-date, If not all)*

Buchert & Ehlers formalism with *"Q"* (Backreaction) as Total Divergence

Static "Lattice of Inhomogeneities" calc's w/only the Final, Clumped "Steady State"

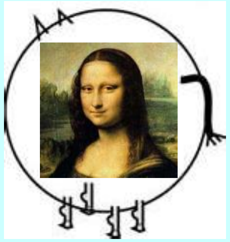
How Now Round Cow? : The Current State of Backreaction Research *(as I see it…)*

Backreaction from Structure Formation, in the **Real Universe**:



The Physics of Backreaction, in most popular models:

Missing *Something*… "Obvious Conclusion": *Milk Does Not Exist!*
(i.e., Backreaction insufficient to generate the observed acceleration)



~~*Still No "Milk"* → **Backreaction Must Fail!**~~

"As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." – A. Einstein

➔ Rather than doing **exact** models of *very approximate* physics, better to do a *very approximate* model of the **exact** physics!

The crucial difference between *Instantaneous* "**Newtonian(-Grav.) Cosmology**", and *Newtonian-strength perturbations* in **General Relativity**, is often overlooked!

Important **caveat** ("the fine-print") from "Averaging inhomogeneous Newtonian cosmologies" (Buchert, T., & Ehlers, J. 1997, A&A 320, 1)

An important difference to the Newtonian treatment, besides spatial curvature, arises due to the fact that it may not be in general possible to represent the term (18) as a divergence in GR. We stress that this would imply a strong challenge for the standard cosmologies, since we can no longer argue, except for non–generic situations, that there exist cases in which the average obeys Friedmann's law. Even more, we don't expect the previously discussed arguments (after eq. (18)) to hold, since the valid theory on the large scales under consideration is general relativity.

(Newtonian-level Backreaction)

…and, from "On average properties of inhomogeneous fluids in general relativity I: dust cosmologies" (Buchert, T., 2000, Gen. Rel. Grav. 32, 105)

We conclude:

3. We were not able to produce an argument analoguous to the Newtonian treatment stating that the 'backreaction term' vanishes for topologically closed space sections, if integrated over the whole space. Without such an argument averaged inhomogeneous cosmologies cannot be identified with the standard FRW cosmologies on any spatial scale. To justify this identification as an approximation there is presently no sufficiently general quantitative result as to whether the 'backreaction' term could be neglected on some scale or, in words suggested by *Corollary 3*, whether the averaged curvature decouples from the inhomogeneities.

Newtonian-Strength Perturbations: Are they **Really** Negligible as a *Total Divergence?*

- Newtonian-level metric perturbations, apparently being expressible in the "Buchert formalism" as a total divergence, are believed to provide (essentially) ***zero*** Backreaction…

➔ Represents a **huge** impediment to acceleration-via-backreaction, since it implies the requirement of *strongly non-Newtonian perturbations* and *relativistic flows!*

From "Averaging inhomogeneous Newtonian cosmologies":
(Buchert, T., & Ehlers, J. 1997, A&A 320, 1)

2. Averages in Newtonian cosmology

According to Newtonian physics, the motion of a self–gravitating, pressureless fluid ("dust") is governed by the *Euler–Poisson system* of equations. Thus, with respect to a non–rotating Eulerian coordinate system[1] the fields of mass density $\varrho(\boldsymbol{x},t) > 0$, velocity $\boldsymbol{v}(\boldsymbol{x},t)$ and gravitational acceleration $\boldsymbol{g}(\boldsymbol{x},t)$ are required to satisfy

$$\partial_t \boldsymbol{v} = -(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} + \boldsymbol{g} \,, \tag{1a}$$

$$\partial_t \varrho = -\nabla\cdot(\varrho \boldsymbol{v}) \,, \tag{1b}$$

$$\nabla\times\boldsymbol{g} = \boldsymbol{0} \,, \tag{1c}$$

$$\nabla\cdot\boldsymbol{g} = \Lambda - 4\pi G\varrho \,, \tag{1d}$$

Maxwell's Equations *(for E& M)*
+ Lorentz Force Law & Continuity Equation
(in analogy with "Newtonian" G.R., for *masses*) :

$$\boxed{\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0}$$

$$\boxed{\boldsymbol{\nabla}\times\boldsymbol{B} = \left(\frac{4\pi}{c}\right)\boldsymbol{J} + (1/c)\ d\boldsymbol{E}/dt}$$

$$\mathbf{f} = \rho\boldsymbol{E} + \boxed{\boldsymbol{J}\times\boldsymbol{B}}$$

Continuity *(Charge/Mass Conservation)*

$$\boldsymbol{\nabla}\times\boldsymbol{E} = \boxed{-(1/c)\ d\boldsymbol{B}/dt}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = 4\pi\rho$$

➢ "Something is Missing"… the *"**Magnetic**" Gravitational Fields!*

…No ***B***-terms ➔ No *Wave Propagation* ➔ **No** *Gravitational Info* From New, *Distant* Structures!

"In the Newtonian approximation the expansion of a domain is influenced by the inhomogeneities inside the domain." (Buchert, Kerscher & Sicka, 2000, Phys. Rev. D62, 043525)

- "**Causal Backreaction**" is the idea that this view is *unacceptable*, even for Newtonian-*Strength* Pert's. !

Vorticity from Virialization: The **Key Factor** vs. Gravity in Stabilizing All Structures

- Vorticity often *deliberately dropped* from calculations! (For convenience…?)

➔ Is Vorticity a "Small-Scale Player"? (e.g., Buchert, T. 2008, Gen. Rel. Grav. 40, 467)
(Relevant only for cosmic averages performed over domains ≤ galaxy cluster scales??)

- The quantity in the Raychaudhuri Equation for the increase of the velocity expansion ($d\theta/dt$) is **vorticity squared (ω^2)**, not the vorticity ($\omega_{\mu\nu}$)…

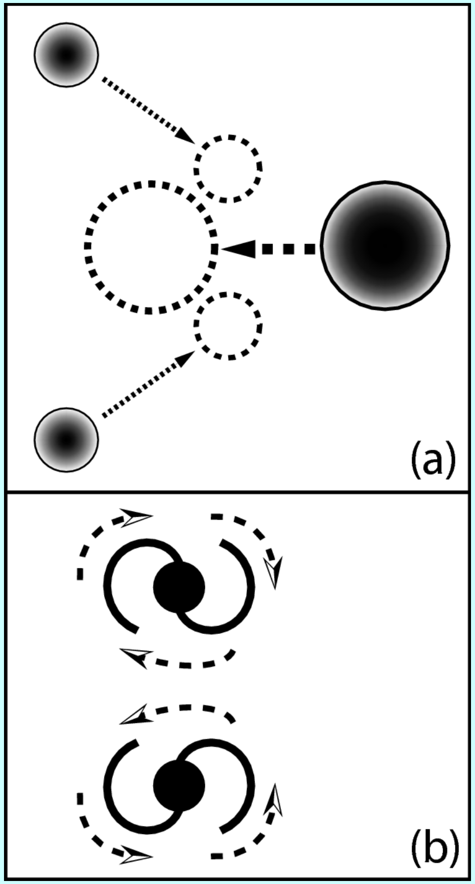
➔ $\langle \omega^2 \rangle$ obviously cannot average away!

$\langle \omega_{\mu\nu} \rangle \approx 0$

$\langle \omega^2 \rangle \approx 0$

$\langle \omega_{\mu\nu} \rangle \approx 0$

$\langle \omega^2 \rangle \neq 0!$

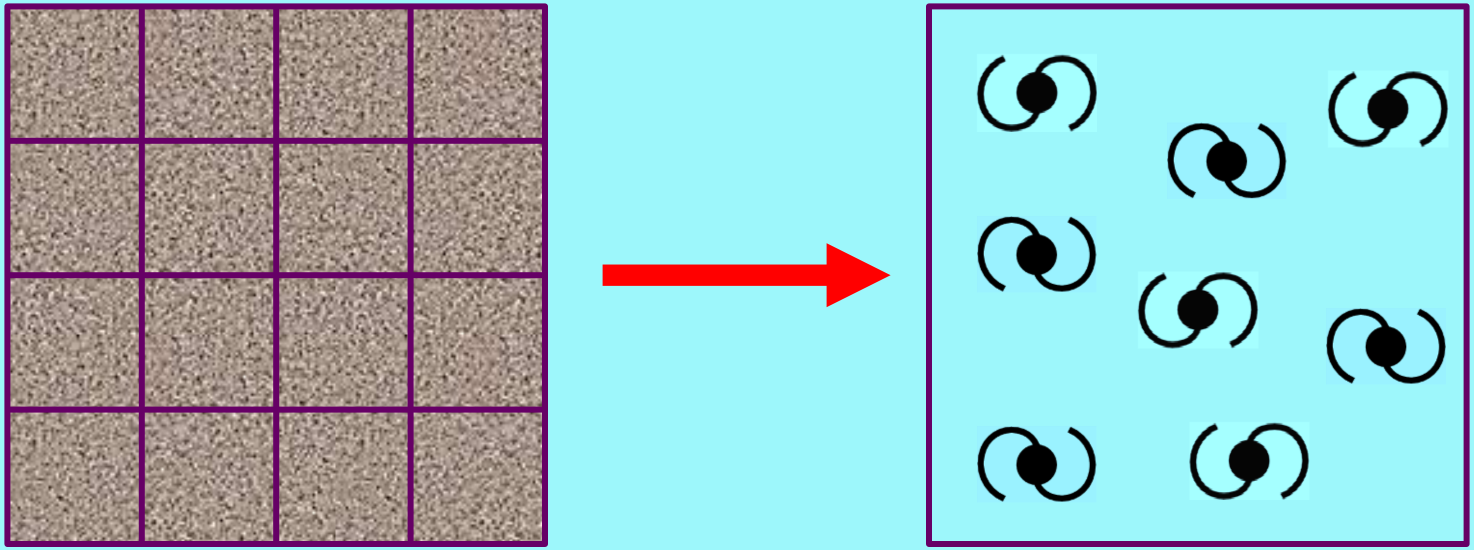


…Cosmic averages of positive semi-definite quantities "attained in the subdomains is 'frozen' and cannot become smaller by averaging over larger domains."
(Buchert & Ehlers, 1997, A&A 320, 1)

∴ Formalisms which *neglect vorticity* as a "small scale player" **cannot** properly estimate the effective backreaction from structure formation!

The Structure-Forming Universe is *defined* by a **key Phase Transition**:

Smooth *(& Little Non-Hubble-Flow Motion)* ➔ Clustered *(& Virialized through Motion)*

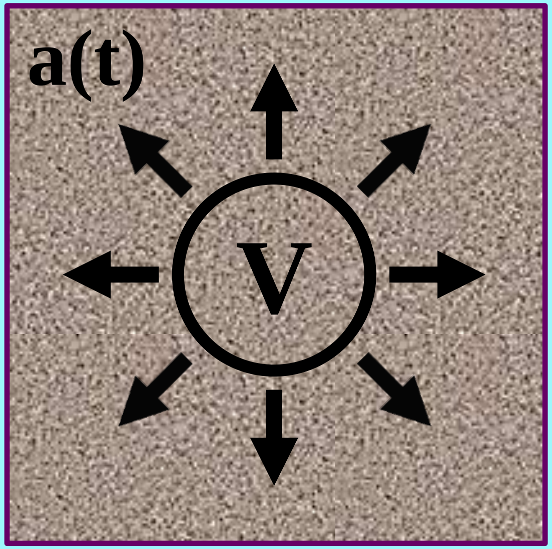


Q: How best to model ***all*** of the relevant physics of this phase transition, without any **exact** (or "complete" perturbative) formalism that captures ***everything?***

A: Use the *relatively simple* nature of the **beginning** and **end** states of the structure-formation process to estimate the net change in the metric, **before ➔ after**.

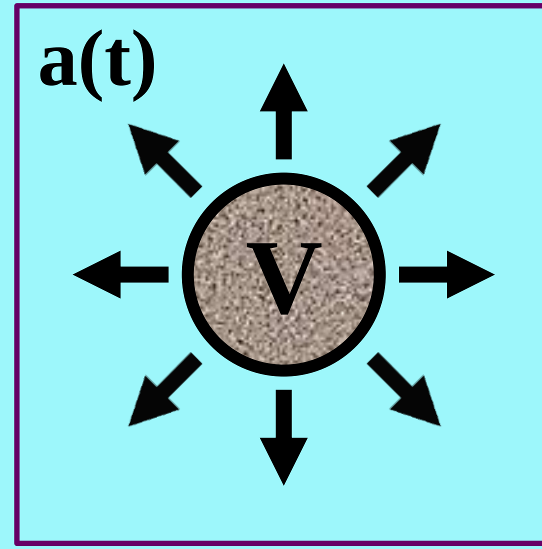
- The **interim dynamics** are less crucial, except to determine the *precise timing;* for now, will constrain this **observationally** as an *empirical* "clumping function".
- Backreaction is a **nonequilibrium** process: it ends when the (causally-observed) structure formation is *complete*. (*"Acceleration"* just a comparison of "then" vs. "now"!)

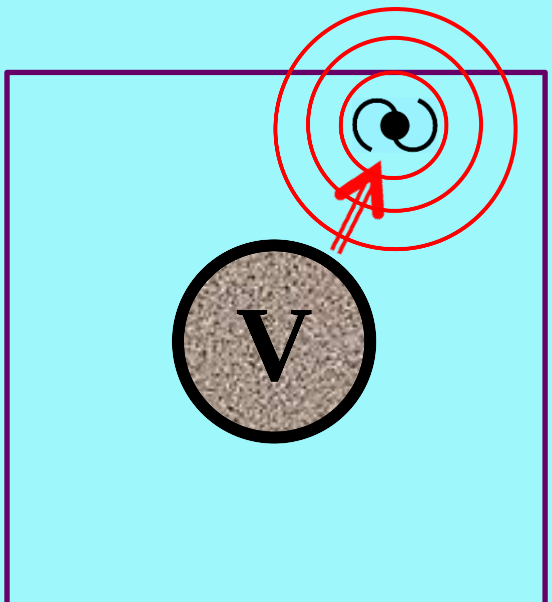
<u>Estimating the *Net Effect* of **Clustering**</u> (on some "local volume", **V**):



As is well known for a <u>homogeneous</u> universe (e.g., Weinberg, 1972, "Gravitation and Cosmology"), the Friedmann expansion for **V** can be derived without reference to *anything* outside of it…

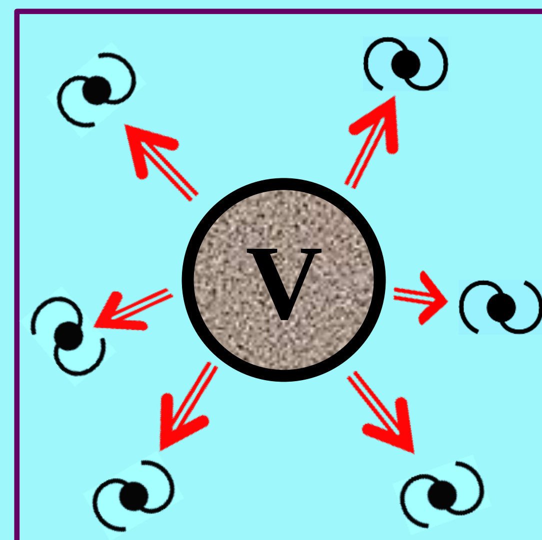
…so "**Remove**" the Exterior! :
(<u>same</u> expansion behavior!)





When the universe becomes <u>inhomogeneous</u>, however, then *individually-clustered* & *vorticity-stabilized* objects become **gravitational attractors**, which pull on all other mass (*including* that within **V**)…

…and these grav. pulls upon **V** are <u>new</u>, as if the objects *"came in from infinity":*



Therefore, can model the **main effect** (upon volume **V**) of Fully-Virialized Clusters by adding in the small, *Newtonian-strength* metric perturbation term for the mass of each self-stabilized system, ***on top of*** the internally-generated FRW metric for **V**:

$$g_{\mu\nu}(\mathbf{V}) = \{Unpert.FRW\} + \sum_{All\ Clumps,\, i} \{-dt^2\,[-2GM_i(\boldsymbol{t})/a(t)r_i] + \underline{dr^2\,a(t)^2[2GM_i(\boldsymbol{t})/a(t)r_i]}\,\}$$

(Must be angle-averaged for clumps in different directions)

➔ Each pert. term *slowly grows from zero* as each mass M_i goes from **smooth** to **fully clumped**.

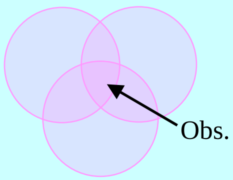
- N.B.: These perturbative factors (esp. the "extra volume" in g_{rr}) are generated only because the clumps do not collapse completely, but *stabilize* themselves w/**vorticity**, velocity dispersion…

- The gravitational *pulls* (forces) from clumps in *different directions* roughly cancel out in **V**; (a "**Smoothly-Inhomogeneous**" Universe); but the Potential Pert's., ΔΦ, *always add together!*

Q: Each individual pert. term is *very small*, $\propto (1/r) \times \{\text{orders } n > 1 \text{ of } (v/c)^n\}$; *can it matter*?

A: Yes, *"little things"* **add up** :

(not like in Swiss-Cheese models!)



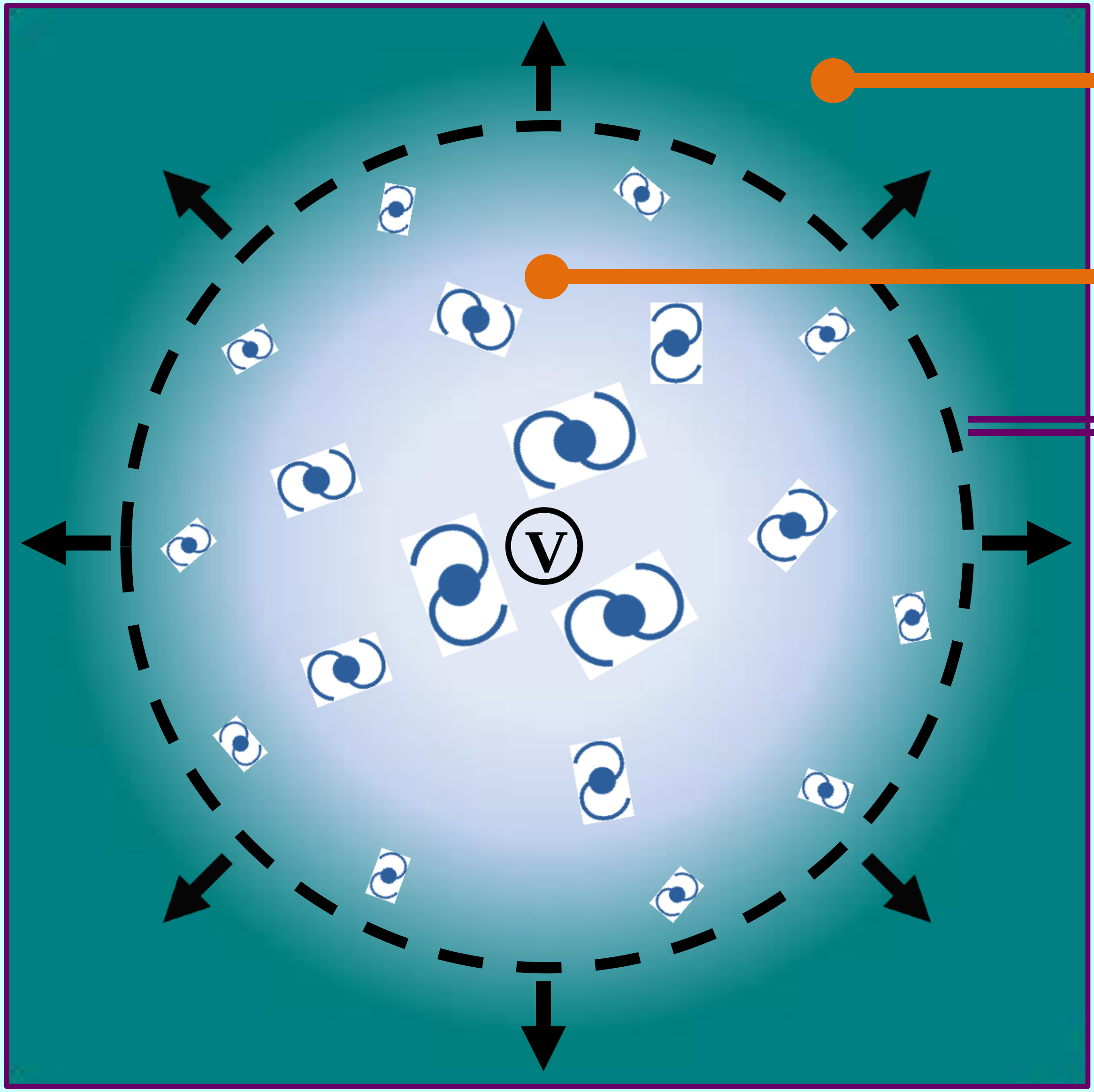
➔ ***Tot. Pert. (at Obs.)*** $\propto \int_{r=0}^{\infty} \frac{r^2 dr}{r} = \infty^2$ **!**

➔ A *stronger divergence* than **Olbers' Paradox**! (…only rendered finite by *causality*…)

Note, for all Perturbation Theory approaches: *"Small Amplitude"* terms **cannot** be reliably neglected, because for cumulative effects, size doesn't matter!

A Simple Analogy: Gravitation is *much weaker* than Electromagnetism; is Gravity "negligible"? Is E&M holding you down in your seat right now?

But What Does an <u>Observer</u> *See*, Considering **Causal** *"Look-Back Times"*?



Universe still *smooth*, **before** onset of clustering

The **later**, *clustered* universe

"<u>Wave of Observed Clumpiness</u>" (the causal edge of clustering obs., *moving outward* at **c** …)

➔"<u>Causal Backreaction</u>" is a **relativistic** process, *even if* most matter obeys $v \ll c$! (**Cannot** drop $O[(v/c)^2]$ terms or time derivatives!)

(Clustered mass density must be evaluated at *retarded time*)

$$\Phi_{\rm SR}(\mathbf{x}, t) = -G \int^{\infty} \frac{[\rho(\mathbf{x}', t)]_{\rm ret}}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

"Causal Updating"

∴ Causal Backreaction effects are <u>finite</u>, though *large!*
(& depend upon <u>faraway</u> pert's., not *local* clustering, as in Pert. Theory)

<u>How to **Implement** Causal Backreaction</u>: (B. Bochner: arXiv:1109.4686 & arXiv:1109.5155)

(1) Choose a "<u>Clumping Evolution Function</u>" to empirically model the time dynamics:

<u>Try</u>: (i) Linear Regime: "ρ_{clump}" ~ $\delta\rho/\rho \sim a(t) \propto \boldsymbol{t^{2/3}}$

(ii) Nonlinear Regime: "ρ_{clump}" ~ $\delta\rho/\rho \sim a(t)^{n\geq 3} \propto \boldsymbol{t^2}$

(iii) Prop. to time for structures to form: "ρ_{clump}" $\propto \boldsymbol{t}$

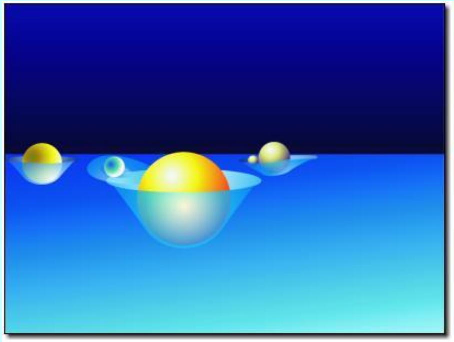
<u>Test</u> all models vs. SNe data, <u>optimizing</u>:

a) $\boldsymbol{t_{init}}$ ≡ "Beginning" of Clustering

b) $\boldsymbol{\Psi_0}$ ≡ Clustered "Mass Fraction" Now

(2) Compute the "**Causal Updating**" integral to get the total metric perturbation, $I(t)$, (at any location) in the past, as a fn. of time:

$$I(t) = \int_0^{\alpha_{\max}(t,t_{\rm init})} \{12\ \Psi[t_{\rm ret}(t,\alpha)]\ [(t_0/t)^{2/3}]\}\ \alpha\ d\alpha$$



(3) Obtain the *final metric* for all <u>Cosmo Calc's</u>:

$$ds^2 = \underbrace{-c^2[1 - I(t)]\ dt^2}_{\text{Slowdown of Observers vs. Cosmic Time } t} + \underbrace{\{[a_{\rm MD}(t)]^2\ [1 + \underline{(1/3)}I(t)]\}\ |d\vec{r}|^2}_{\text{(Some) Volume Creation}}$$

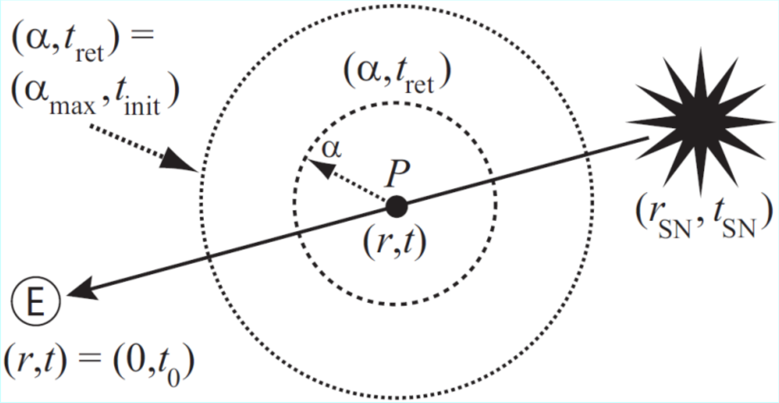
3D angle-averaging

Slowdown of Observers vs. Cosmic Time *t*

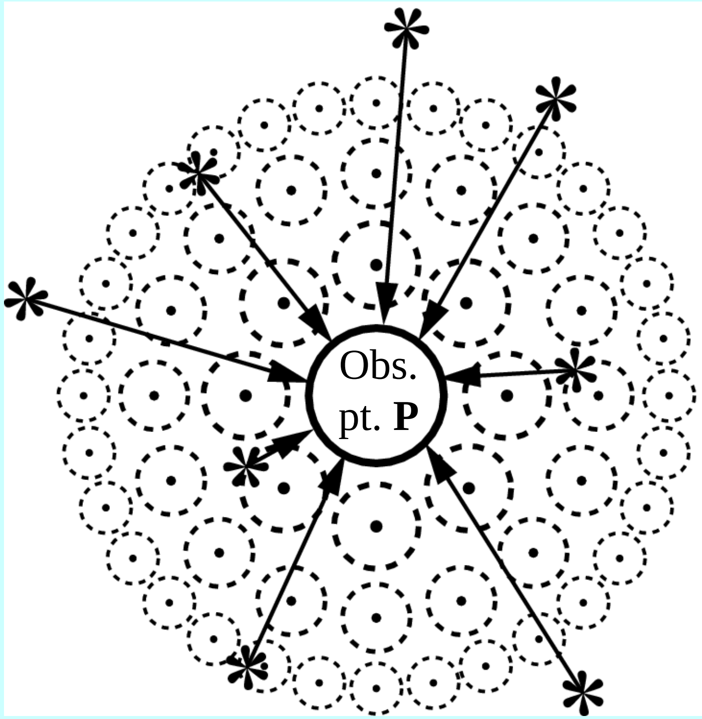
(Some) Volume Creation

Sufficient for ***Apparent*** **Acceleration!**

(4) Integrate the trajectory of a SN light-ray to calculate **Luminosity Distance** as a fn. of *z*, & thus get *many other* <u>Cosmological Params.</u>:



Important Complication: *Old* metric perturbations from structures **slows down** all *new*, *ongoing* propagation of inhomogeneity information – Weakening "Causal Updating"!



- Nothing other than **Shapiro time delays** on all propagating light & gravitational information.

- Accumulates over time… **Causal** Backreaction has a *negative feedback loop upon* itself…!
  - *"Eternal"* acceleration not likely here…

- This behavior is *recursive* – later metric pert. effects depend upon prior ones, for a *nonlinear* response to clustering – so call this "**Recursive Nonlinearities**" (to distinguish it from *nonlinear Gen. Rel.* effects).

Version "Cow 2.0",
now **including**
Recursive Nonlinearities:

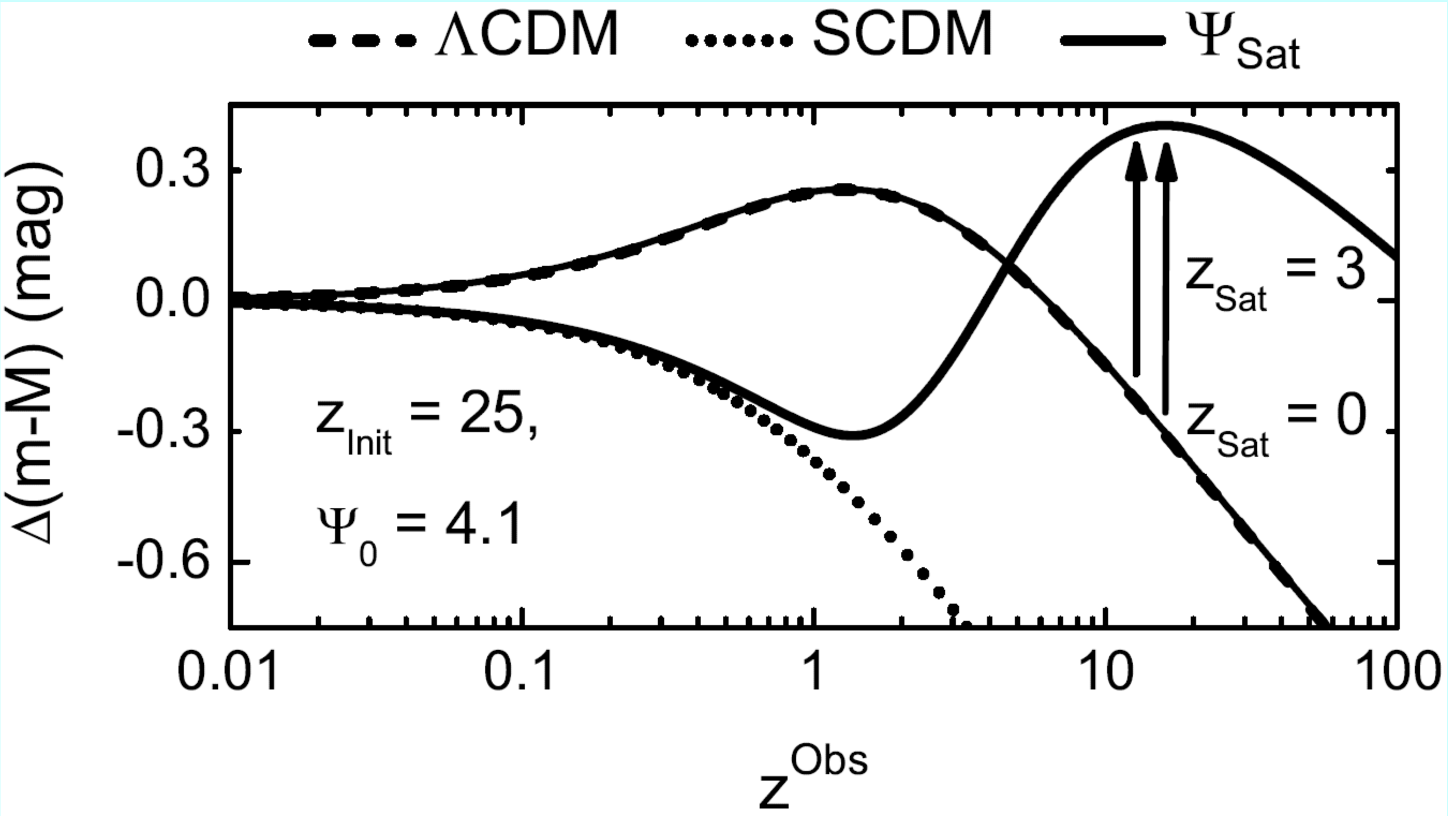
(B. Bochner: arXiv:1206.5056)

$$\alpha_{\max,i} = \alpha_{\max,(i-1)} + \left\{ \frac{1}{3} \frac{\sqrt{1 - I^{\mathrm{RNL}}_{(i-1)}}}{\sqrt{1 + [I^{\mathrm{RNL}}_{(i-1)}/3]}} \frac{\Delta T}{[T_{(i-1)}]^{2/3}} \right\} ,$$

$$X_i^{\mathrm{RNL}} = \frac{12}{T_i^{2/3}} \sum_{k=\{1,(i-2)\}} \{ \Psi[T_{(i-k)}][\alpha_{\max,i} - \alpha_{\max,(i-k)}][\alpha_{\max,(i+1-k)} - \alpha_{\max,(i-k)}] \}$$

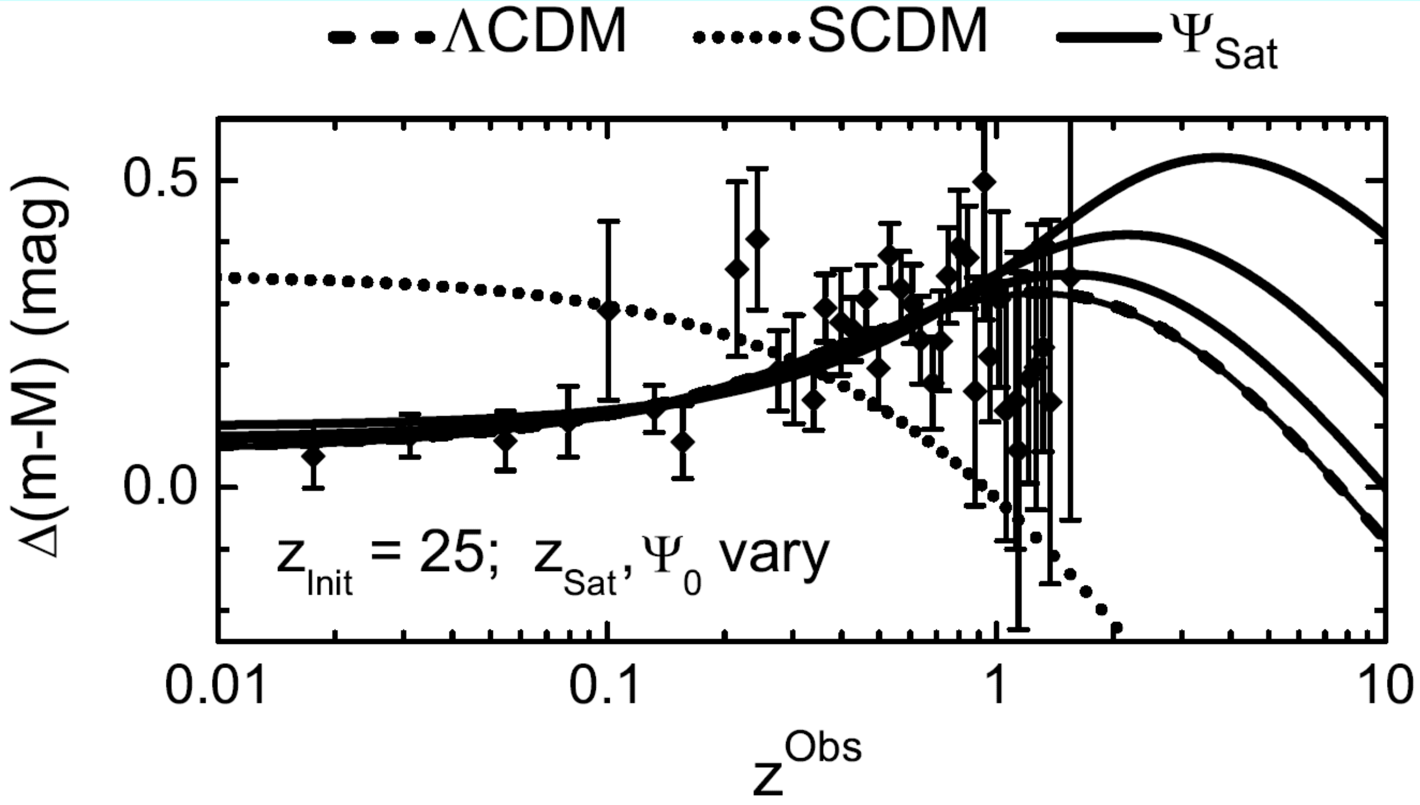
$$I_i^{\mathrm{RNL}} \sqrt{1 + [I_i^{\mathrm{RNL}}/3]} = X_i^{\mathrm{RNL}} .$$

With Recursive Nonlinearities: *Too Much* Early Causal Backreaction **Slows Down** Later Effects!



- Causal Backreaction models **can** successfully mimic LCDM "Acceleration"! (…as long as structure formation & backreaction occur *gradually* enough…)
  - ➢ Models from the Nonlinear Clustering Regime (% Clump $\propto t^2$) are preferred
  - ➢ But due to self-limiting feedback, **no** *"Big Rip"* likely from Causal Backreaction

Causal Backreaction models can reproduce the Apparent Acceleration *(Union Type Ia SNe)* for a "Smoothly-Inhomogeneous" Universe, without Dark Energy, Voids, etc.:



- Though successful at producing an "acceleration", need *large* clustering ($\Psi_0 \sim 2-4$)
  - ➢ Equivalent to nearly-complete clustering on several different "hierarchical" scales simultaneously… (Stellar Clusters, Galaxies, Galaxy Clusters, etc… all individually stabilized through virialization)

- Less final clustering OK if using models where structure formation "saturates" late…
(due to clumping-inhibiting feedback from "gastrophysics", and/or from the "acceleration" itself…)
  - ➢ Consistent with Vikhlinin, et al. (arXiv:0812.2720) results on recent effects on the growth of clustering

Final Results for "**Best-Fitting**" models found (even without a rigorous optimization):

Table 3: Output Cosmological Parameters from our RNL Runs with 'Early Saturation'

z_{Sat}	$\Psi_{0,\mathrm{Opt}}$ [a]	χ^2_{Fit}	P_{Fit} [b]	I_0	z^{Obs}	H_0^{Obs}	H_0^{FRW}	t_0^{Obs}	$\Omega_{\mathrm{M}}^{\mathrm{FRW}}$	w_0^{Obs}	j_0^{Obs}	$l_{\mathrm{A}}^{\mathrm{Obs}}$
				Ψ_{Sat} Clumping Model Runs, $z_{\mathrm{init}} = 25$								
0	4.1	311.8	0.351	0.53	1.14	70.07	42.32	13.64	0.943	-0.751	1.73	294.5
0.25	2.6	313.5	0.326	0.58	1.15	69.60	40.24	14.00	1.054	-0.620	0.15	289.7
0.5	2.3	316.6	0.284	0.68	1.15	69.40	36.32	14.65	1.338	-0.585	-0.14	279.8
1	2.2	320.2	0.238	0.80	1.14	68.77	29.54	15.75	2.086	-0.488	-0.94	259.9
	Comparison Values from Best-Fit flat ΛCDM Model ($\Omega_\Lambda = 0.713 = 1 - \Omega_{\mathrm{M}}$)											
...	...	311.9	0.380	...	1.0	69.96	69.96	13.64	0.287	-0.713	1.0	285.4

- Causal Backreaction models **fit** the (Union) **SNe data** essentially as well as flat ΛCDM
- A *sufficiently powerful* **Apparent Acceleration** now exists, to be consistent with important complementary Cosmological data sets
- There is a *significant difference* between the "bare" Hubble Constant ($H_0^{\mathrm{FRW}} \propto 1 / t_{\mathrm{FRW}}$), and the "dressed" Hubble Constant actually **observed** (H_0^{Obs}) ; . . . *And* ***so***…
- Since the calculated "Critical Density" goes like $\rho_{\mathrm{Crit}} \propto H_0^2$, and $H_0^{\mathrm{FRW}} \ll H_0^{\mathrm{Obs}}$, so that an *initially flat* matter-only universe ($\Omega_{\mathrm{M}}^{\mathrm{FRW}} \equiv 1$) now **looks** under-dense ($\Omega_{\mathrm{M}}^{\mathrm{Obs}} \sim 0.3$) !
- Other Cosmological Parameters also become Concordant without Dark Energy, such as the Observed Age of the Universe (t_0^{Obs}) and the CMB 1st Peak Position ($l_{\mathrm{A}}^{\mathrm{Obs}}$)
- Deviations from ΛCDM can be tested via $j_0^{\mathrm{Obs}} \neq 1$ (though no iron-clad prediction yet)